ETP/GDOP Behavior Study for N-Sensors Arrays in a Multilateration Radar System.

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Abstract–In this paper, we evaluated the ETP (Expected Theoretical Precision) and GDOP (Geometric Dilution Of Precision) enhancement related to the number of sensors in a Multilateration radar system. An introduction about the principles of the Multilateration radar system basis operation is described, then, the formulation for evaluation the ETP/GDOP of the 3D positioning is shown. We observed that the ETP and GDOP enhance with the increase of the number of sensors. A substantial improvement was obtained until nine sensors but, for more sensors that improvement is reduced. Results for a 75km×75km area are shown, including LAM (Local Area Multilateration) and WAM (Wide Area Multilateration) settings and different values of the aircraft height (5m for LAM surface, 5000m and 8000m for WAM). Additional parameters are shown in order to evaluate the system quality. These parameters are the Expected Theoretical Precision Gain (ETP Gain), Homogenization Level (HL) and Percentage Over a Reference Value (PORV). Due to the proportionality between the ETP and GDOP, only ETP results are shown. In the simulations the same settings of the sensors was used; 3ns for instrumental error and 27m for antenna height. These are typically values for real Multilateration radar system used for the air traffic control.

Keywords– Expected Theoretical Precision, Geometric Dilution of Precision, Multilateration.

I. INTRODUCTION

The ETP describes the magnitude of the position estimation error and, the GDOP describes the effect of the geometry in the relation between position estimation error and the measurement error [1, 2]. The ETP is a dimensional quantity that depends on the measurement types and, usually is expressed in meters. On the other hand, the GDOP is a dimensionless quantity.

The Multilateration systems use TDOA (Time-Difference Of Arrival) measurements to calculate an hyperbolic positioning in order to estimate the location. Three measurements are required to estimate the position and two times of arrival (TOA) to obtain a measure. Therefore, at least four sensors are needed to get a hyperbolic positioning.

The quality of hyperbolic positioning expressed by the ETP and the GDOP depends on multiple factors, such as thermal, instrument and propagation errors [2]; and number of sensors. This last factor directly affects the Multilateration system cost; therefore, a maximum coverage with minimum number of sensors for the same system quality is desirable.

When designing such a Multilateration system, it is useful to know the minimum number of sensors that estimate the position with the statutory precision.

Simulations have been carried out to evaluate the ETP/GDOP enhancement related to the number of sensors. According to [2, 3], the best geometric distribution is a polygon where sensors are equally spaced. A 3D positioning distribution is used with all sensors equally spaced into a circumference with radio r and one of them located in the center.

In the second part, ETP/GDOP equations, relations and their means are shown. In the third part, only ETP results are shown, since ETP and GDOP are proportionally direct and their graphics show the same behavior. As well, Homogenization Level and Percentage Over a Reference Value results are shown for values of Expected Theoretical Precision of 15m, 10m and 7m.

II. EXPECTED THEORETICAL PRECISION (ETP) / GEOMETRIC DILUTION OF PRECISION (GDOP)

Defining sensor's position by the vector:

$$S_i = [x_i \ y_i \ z_i] \tag{1}$$

And coordinates of a space point by

$$S_i = [x_i \ y_i \ z_i] \tag{2}$$

The ETP is defined by [1]:

$$\text{ETP} = \sqrt{\text{trace}\left[c^2 (F^T H^T N^{-1} H F)^{-1}\right] \ (m)} \tag{3}$$

Where trace(A) denotes the sum of the principal diagonal of matrix A. Parameter c is the vacuum speed of light (m/s), F is the matrix of geometry, N the difference matrix of the instrumental error covariance matrix of the sensors and, H is the matrix of differences. F, N and H are defined by:

$$F = \begin{bmatrix} \frac{(R_o - S_1)^T}{D_{o1}} \\ \vdots \\ \vdots \\ \frac{(R_o - S_n)^T}{D_{on}} \end{bmatrix}_{n\times 3} \tag{4}$$

n is the system number of sensors and D_{on} is the distance between sensor n and measure point.

$$N = H N_\epsilon H^T|_{(n-1)\times(n-1)} \tag{5}$$

$$N_\epsilon = \begin{bmatrix} {\sigma_1}^2 & 0 & \cdots & \cdots & 0 \\ 0 & {\sigma_2}^2 & 0 & \cdots & \vdots \\ \vdots & 0 & {\sigma_3}^2 & 0 & \vdots \\ \vdots & \vdots & 0 & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & {\sigma_n}^2 \end{bmatrix}_{n\times n} \tag{6}$$

σ_n is sensor-n instrumental error (rms)

According with [1], the GDOP is defined by:

$$GDOP = \frac{ETP}{c\sigma_s} \tag{7}$$

$c\sigma_s$ is the system mean-square error, where

$$\sigma_s = \sqrt{\frac{1}{n}\sum_{i=1}^{n} {\sigma_i}^2} \tag{8}$$

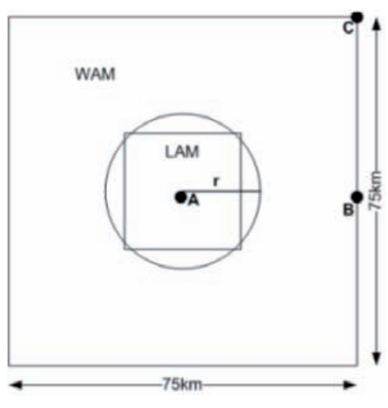
The ETP gives information about the precision of the position estimation in a point of space, for a fixed sensor geometric configuration and instrumental errors. This parameter is very helpful for system reliability studies in order to know precision degradation due to fails of some elements in the system.

The GDOP describes how much affects sensors geometry to position estimation. Because it is a dimensionless quantity, its numeric results have been interpreted by a qualitative scale that describes from an optimum to a bad estimation. This scale depends on the environment operational system. If it is LAM (Local Area Multilateration), it will have a correspondence with numeric values and if it is WAM (Wide Area Multilateration) it will have another correspondence. Generally the precision required for LAM is higher than the one required for WAM. The GDOP is very helpful for system design.

III. RESULTS

Figure 1. shows a 75km×75km area over which simulations were done. Also, it shows a circumference with radio r where sensors were located. The area has been divided into two zones, one for LAM and other for WAM.

FIG. 1. SIMULATION AREA



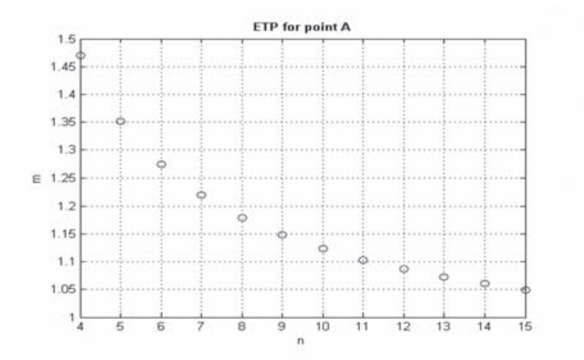
Fuente: Los autores

Instrumental error of 3ns and antenna height of 27m were used for all sensors.

A. ETP evaluation for points A, B y C.

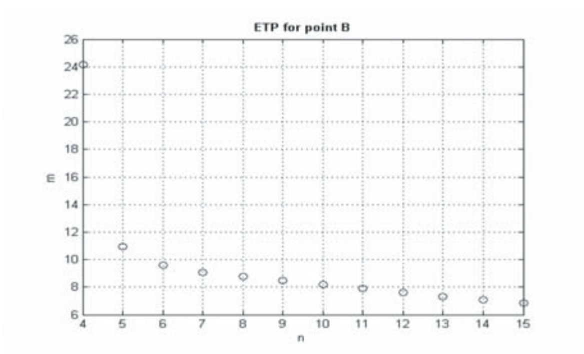
Point A height is 5m (LAM, surface) , for point B is 5000m (WAM) and for point C is 8000m (WAM).

FIG. 2. ETP FOR POINT A



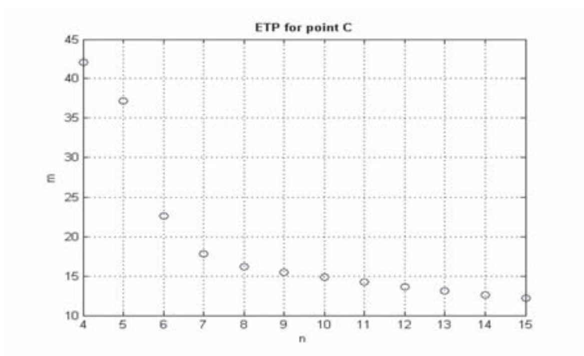
Fuente: Los autores

FIG. 3. ETP FOR POINT B



Fuente: Los autores

FIG. 4. ETP FOR POINT C.



Fuente: Los autores

Figures 2-4 show that the ETP enhance with the number of sensors (lower ETP). However, for more than nine sensors the precision increment is stabilized and the obtained values do not justify the increment in the number of sensors.

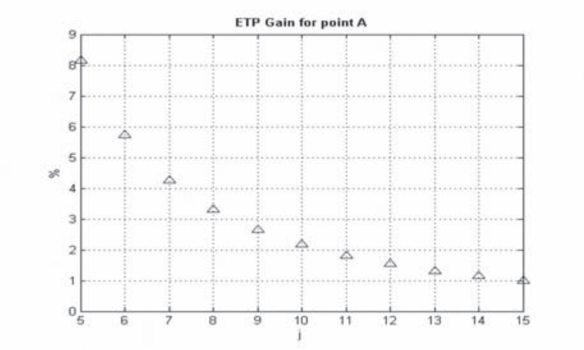
In order to validate the above affirmation, ETP gain (or precision gain) for using sensors or sensors is defined by:

ITECKNE Vol. 6 Número 1 • Junio 2009 • 29 - 34

$$\text{Gain}_{ij} = \frac{ETP_i - ETP_j}{ETP_i} \times 100 \quad (\%) \qquad (9)$$

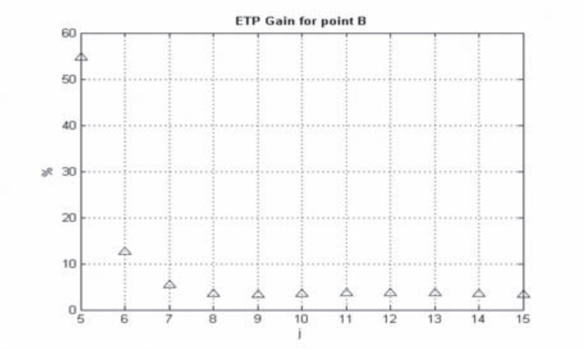
Figures 5-7 show for points A, B and C.

FIG. 5. ETP GAIN FOR POINT A



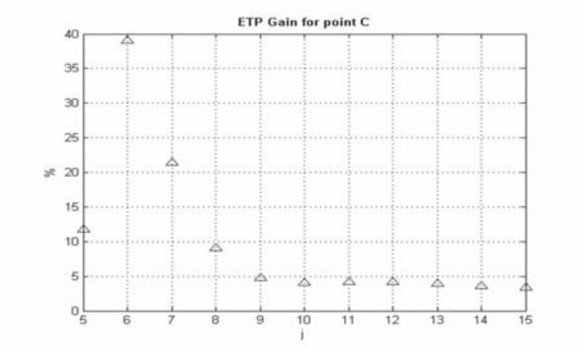
Fuente: Los autores

FIG. 6. ETP GAIN FOR POINT B



Fuente: Los autores

FIG. 7. ETP GAIN FOR POINT C

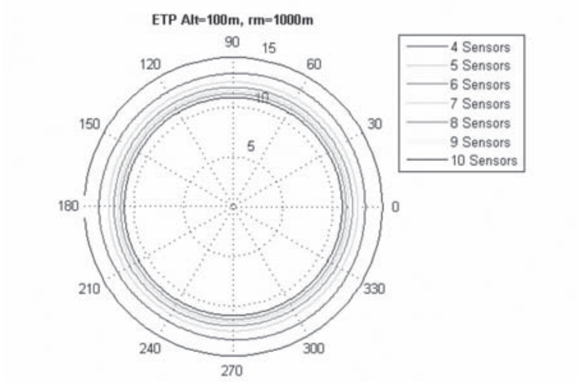


Fuente: Los autores

A minimum ETP gain is observed in point A (figure 5) because this is the optimum point of the evaluated area [2]. For point B, 66% of enhance is obtained when increasing from four to nine sensors. Finally, for point C 62% ETP gain is obtained.

B. ETP evaluation over a circumference of radio rm

FIG. 8. ETP FOR THE CIRCUMFERENCE WITH RM=1000M AND HEIGHT=100M.



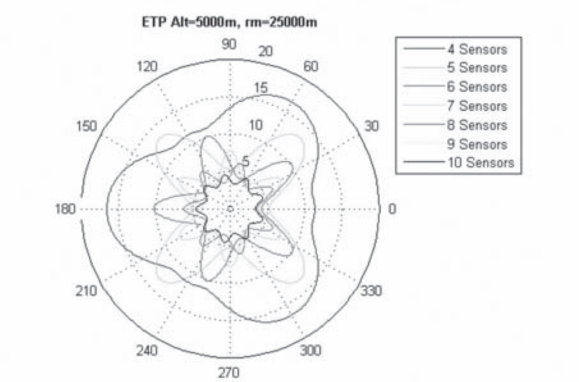
Fuente: Los autores

In figure 8 is observed that precision is enhanced by the number of sensors in the system, but with a low gain. The reason is due to rm=1000m is a LAM perimeter very near to central point (optimum point).

In figures 9 and 10, an enhance precision is observed when number of sensors increase in the system, with a high ETP gain. In these cases, a fluctuation in a range of values can be seen. This fluctuation is equally minimized when the increment of number of sensors.

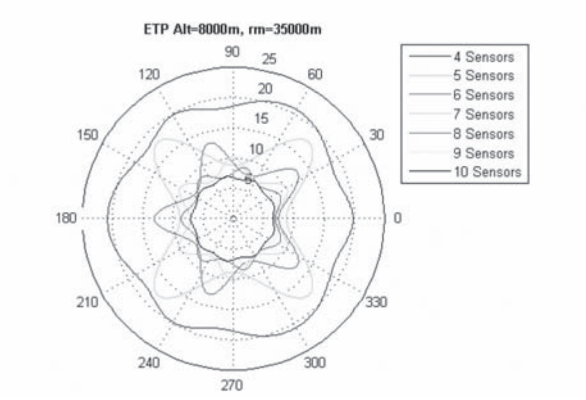
For nine sensors, similar to part A, a stabilization in the ETP gain is observed and for ten sensors the enhance grade is very small.

Figrua 9. ETP FOR THE CIRCUMFERENCE WITH RM=25000 AND HEIGHT=5000M.



Fuente: Los autores

FIG. 10. ETP FOR CIRCUMFERENCE RM=35000M AND HEIGHT=8000M



Fuente: Los autores

A homogenization level (HL) in the precision of a system with i sensors is defined as

$$HL_i = \frac{ETP_{min}}{ETP_{max}} \tag{10}$$

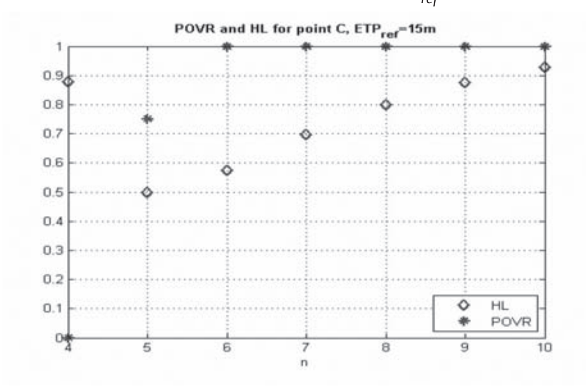
And the Percentage Over a Reference Value for the ETP of a system with sensors is defined as:

$$PORV_i = \frac{Total\ points\ \{ETP \leq ETP_{ref}\}}{Total\ points} \tag{11}$$

The Homogenization Level indicates the fluctuation minimization of the Expected Theoretical Precision when the number of sensors increases. The Percentage Over a Re ference Value shows the covered points percentage with a precision stipulated.

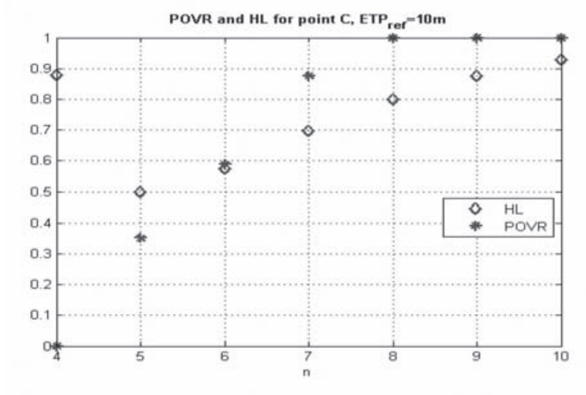
The optimal system is the one that has PORV=1 and HL=1. In figures 11-13, PORV and HL for point C with ETP_{ref} of 15m, 10m and 7m are shown respectively.

FIG. 11. POVR AND HL FOR POINT C, $ETP_{ref} = 15\text{m}$



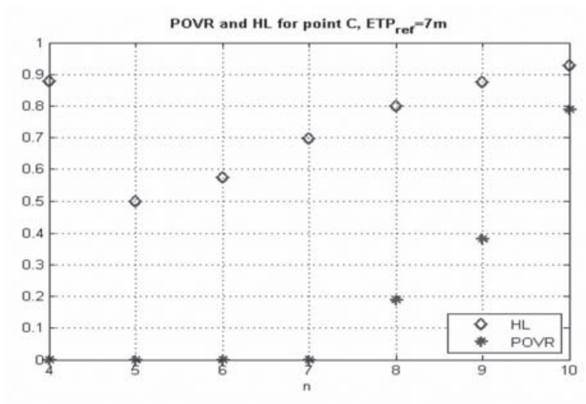
Fuente: Los autores

FIG. 12. POVR AND HL FOR POINT C, $ETP_{ref} = 10\text{m}$



Fuente: Los autores

FIG. 13. POVR AND HL FOR POINT C, $ETP_{ref} = 7\text{m}$



Fuente: Los autores

CONCLUSIONS

It has been evaluated the enhancement of ETP/GDOP for a Multilateration system, getting a precision rise when increasing the number of sensors in the system. Nine sensors is the maximum number of sensors to obtain a significant precision gain, from ten sensors on, the gain obtained does not justify the inclusion of more sensors in the system.

The Expected Theoretical Precision/Geometric Dilution of Precision study is very helpful to know the reliability and service degree of a Multilateration system such in LAM environments as in WAM environments. This study gives information about the minimum number of sensors that could be used to maintain an adequate quality level in the aircraft positioning.

The increase in the number of sensors means higher coverage with better precision and more homogenization, which gives more reliability to the system. Also is sufficient up to nine sensors in order to generate a higher grade of reliability to the system.

The ETP Gain, Homogenization Level and Percentage Over a Reference Value results obtained are very useful in the optimization processes for Multilateration system deployment. These parameters include information that depends on all variables involve in the system functionality.

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